NOM :

INTERRO DE COURS – SEMAINE 3

Exercice 1 – On considère les matrices

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} \quad \text{et} \quad B = \begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix}.$$

Calculer $B - A$, $A + 2B$, $-(A - 2I_3)$ et $I_3 + A + B$.

Solution :

$$B - A = \begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix} - \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 1 & 3 \\ -1 & -4 & -6 \\ 2 & 3 & -4 \end{pmatrix}$$

$$A + 2B = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 1+0 & 0+2 & -2+2 \\ 0-2 & 2-4 & 3-6 \\ 1+6 & -1+4 & 0-8 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -2 & -3 \\ 7 & 3 & -8 \end{pmatrix}$$

$$-(A - 2I_3) = -\left(\begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} - 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right) = -\begin{pmatrix} 1-2 & 0-0 & -2-0 \\ 0-0 & 2-2 & 3-0 \\ 1-0 & -1-0 & 0-2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -3 \\ -1 & 1 & 2 \end{pmatrix}$$

$$I_3 + A + B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 1 \\ -1 & -2 & -3 \\ 3 & 2 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 0 \\ 4 & 1 & -3 \end{pmatrix}$$

Exercice 2 – Effectuer les produits suivants.

1. $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \\ 0 & 6 \end{pmatrix} \times \begin{pmatrix} 10 & 1 \\ 0 & -3 \end{pmatrix}$
2. $B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ -1 & 5 & 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$
3. $C = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 4 & 7 \\ 8 & -3 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 & 4 \\ -1 & 0 & 3 \\ -3 & 1 & \sqrt{2} \end{pmatrix}$
4. $D = \begin{pmatrix} 1 & 2 & -4 \\ 4 & 6 & -3 \\ 2 & -5 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Solution :

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \\ 0 & 6 \end{pmatrix} \times \begin{pmatrix} 10 & 1 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 10+0 & 1-6 \\ 30+0 & 3+12 \\ 0+0 & 0-18 \end{pmatrix} = \begin{pmatrix} 10 & -5 \\ 30 & 15 \\ 0 & -18 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ -1 & 5 & 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2+4+2 \\ 6+2+0 \\ -2+10+0 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ 8 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 4 & 7 \\ 8 & -3 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 & 4 \\ -1 & 0 & 3 \\ -3 & 1 & \sqrt{2} \end{pmatrix} = \begin{pmatrix} 2-2-9 & 1+0+3 & 4+6+3\sqrt{2} \\ -2-4-21 & -1+0+7 & -4+12+7\sqrt{2} \\ 16+3-6 & 8+0+2 & 32-9+2\sqrt{2} \end{pmatrix}$$
$$= \begin{pmatrix} -9 & 4 & 10+3\sqrt{2} \\ -27 & 6 & 8+7\sqrt{2} \\ 13 & 10 & 23+2\sqrt{2} \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 2 & -4 \\ 4 & 6 & -3 \\ 2 & -5 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+2y-4z \\ 4x+6y-3z \\ 2x-5y+z \end{pmatrix}$$